# An Introduction to Computational Stochastic PDEs, CUP, 2014

Here are the typos/errors that we know about in the first edition. Detailed corrections to MATLAB codes are given on-line [1]. Let us know if you find any more.
1st August 2026

Chapter 1. **p8** In Figure 1.2(b), the assertion that $u(x) = x^{-2}$ belongs to $L^p(1,\infty)$ for $p > 2$ is not optimal. Since $\int_1^\infty x^{-2p}\,dx$ is finite whenever $p > 1/2$, we have $u \in L^p(1,\infty)$ for every $p \geq 1$.

**p10** In the definition of $L^2(\Omega, L^2(D))$, the condition should read $u(\cdot,\omega) \in L^2(D)$ for almost every $\omega \in \Omega$ (not $\omega \in D$).

**p17** In Definition 1.60, the Hilbert–Schmidt norm should be defined as

$$\|L\|_{\mathrm{HS}(U,H)} := \left(\sum_{j=1}^{\infty} \|L\phi_j\|^2\right)^{1/2},$$

(the norm on the right is the one in $H$ not in $U$).

**p21** Lemma 1.78 (Dini's lemma) requires additionally the assumption that $f$ is continuous (to guarantee that $f(x) - f_n(x) \geq \epsilon$ for the limit point $x$). In the application of Dini's lemma for the proof of Theorem 1.80 (Mercer's theorem), this holds true for $g(x) = G(x,x)$.

**p23, p24** In Examples 1.83 and 1.84, the eigenvalue problems are printed with $\frac{d\phi_j^2}{dx^2}$ (resp. $\frac{d\phi^2}{dx^2}$). These should read $\frac{d^2\phi_j}{dx^2}$ (resp. $\frac{d^2\phi}{dx^2}$).

**p29** In (1.33), the first identity requires the conjugate transpose and should read

$$\widehat{\boldsymbol{u}}^*\widehat{\boldsymbol{v}} = d\,\boldsymbol{u}^*\boldsymbol{v},$$

where $^*$ denotes the conjugate transpose. As printed, with the transpose $^\mathsf{T}$, the identity fails even for real vectors: for $d = 4$, $\boldsymbol{u} = [1,1,0,0]^\mathsf{T}$ and $\boldsymbol{v} = [1,-1,0,0]^\mathsf{T}$ give $\boldsymbol{u}^\mathsf{T}\boldsymbol{v} = 0$ but $\widehat{\boldsymbol{u}}^\mathsf{T}\widehat{\boldsymbol{v}} = 4$. The accompanying identity $\|\widehat{\boldsymbol{u}}\|_2 = \sqrt{d}\,\|\boldsymbol{u}\|_2$ is correct.

**p29** For real data $\boldsymbol{u} \in \mathbb{R}^d$, the symmetry of the DFT should read $\hat{u}_\ell = \overline{\hat{u}_{d+2-\ell}}$ (indices modulo $d$), not $\hat{u}_\ell = \overline{\hat{u}_{d-\ell}}$. The latter is correct only under the convention $\ell = 0,\dots,d-1$, whereas the book indexes $\ell = 1,\dots,d$. For example, $d = 4$ and $\boldsymbol{u} = [1,2,3,4]^\mathsf{T}$ give $\widehat{\boldsymbol{u}} = [10, -2+2\mathrm{i}, -2, -2-2\mathrm{i}]^\mathsf{T}$, so that $\hat{u}_2 = \overline{\hat{u}_4}$.

**p32** In the example following Lemma 1.100, the sentence should read that $\|I_J u\|_{L^2}$ and $\|I_J u\|_{H^1}$ provide estimates of $\|u\|_{L^2}$ and $\|u\|_{H^1}$ (not of $\|u\|_{H^1}$ and $\|u\|_{H^2}$), in agreement with the displayed equations that follow and with the caption of Figure 1.5.

**p34** In Theorem 1.107 (Hankel transform), the constant in the case $d = 3$ should be $\sqrt{2/\pi}$ and not $\sqrt{2\pi}$:

$$\hat{u}(s) = \sqrt{\frac{2}{\pi}} \int_0^\infty \frac{\sin(rs)}{rs}\, u(r)\, r^2\, dr.$$

This follows from the proof, which gives $\int_{\mathbb{R}^3} \mathrm{e}^{-\mathrm{i}\boldsymbol{\lambda}^\mathsf{T}\boldsymbol{x}} u(\boldsymbol{x})\, d\boldsymbol{x} = 4\pi \int_0^\infty \frac{\sin(rs)}{rs} u(r) r^2\, dr$, together with $4\pi/(2\pi)^{3/2} = \sqrt{2/\pi}$. It is also consistent with the case $d = 1$.

**p34** In Example 1.105, the sentence should read that the Fourier transform of the derivative equals $\mathrm{i}\lambda$ times the Fourier transform (not "the derivative of the Fourier transform"), in agreement with the preceding display $\widehat{u'}(\lambda) = \mathrm{i}\lambda\hat{u}(\lambda)$.

**p38** In Exercise 1.23, the projection $P_J$ should be onto $\operatorname{span}\{\mathrm{e}^{\mathrm{i}jx} \colon j = -J/2+1,\dots,J/2\}$ and not onto $\operatorname{span}\{\mathrm{e}^{2\pi\mathrm{i}jx}\}$. On $(0,2\pi)$ the functions $\mathrm{e}^{\mathrm{i}jx}$ are the eigenfunctions of $Au = u - u_{xx}$ and the basis used in (1.35). The printed functions have period 1 and are not even orthogonal on $(0,2\pi)$, since $\int_0^{2\pi} \mathrm{e}^{2\pi\mathrm{i}mx}\,dx = \left(\mathrm{e}^{4\pi^2\mathrm{i}m} - 1\right)/(2\pi\mathrm{i}m) \neq 0$ for integers $m \neq 0$.

[1] `https://github.com/tonyshardlow/picspde`

**p38** In Exercise 1.23(c), the final rate should read

$$\|u_1\|_{H^1(0,2\pi)} - \|I_J u_1\|_{H^1(0,2\pi)} = \mathcal{O}(J^{-4}).$$

The periodic extension of the function $u_1$ of Figure 1.6 is $C^2$: $u_1$, $u_1'$ and $u_1''$ all agree at the two ends and only $u_1'''$ jumps, so three integrations by parts are available and $u_n = \mathcal{O}(n^{-4})$, giving $p = 3$. The exercise is therefore in the case $2r < p$ of the first part, not the borderline $2r = p$ that produces $\mathcal{O}(J^{-3} \log J)$. On $(0, 1)$ the coefficients are exactly $3/(2\pi^4 n^4)$.

**p38** In Exercise 1.23(c), $u_1$ is defined on $(0, 1)$ by Figure 1.6, whereas the exercise works on $(0, 2\pi)$; $u_1$ should be read as the corresponding rescaled function. The rates are unaffected.

**p39** In Exercise 1.28, "Making use of DCT-1, evaluate $u^N(x)$ at $x = (k-1)\pi/(N-1)$" should read $v^N(x)$, the function defined immediately above and plotted in the same sentence.

Chapter 2. **p42** Figure 2.1 is misprinted.

**p56** In the first displayed equation of the proof, the first term in the integrand should be $p(x)(e(x)')^2$ (the $e(x)$ has one too many dashes).

**p62** In the proof of Theorem 2.43, the displayed bound following (2.60) should read

$$|u_0|_{H^1(D)} \le a_{\min}^{-1}\Big(K_{\mathrm{p}}\|f\|_{L^2(D)} + a_{\max}K_\gamma\|g\|_{H^{1/2}(\partial D)}\Big),$$

with the constant $K_\gamma$ present in the second term. This is what (2.60) gives on dividing by $|u_0|_{H^1(D)}$, and it is needed for the constant $K := \max\{K_{\mathrm{p}}a_{\min}^{-1},\ K_\gamma(1 + a_{\max}a_{\min}^{-1})\}$ stated in the theorem: combining it with $|u_g|_{H^1(D)} \le K_\gamma\|g\|_{H^{1/2}(\partial D)}$ from (2.54) reproduces exactly that $K$.

**p79** The right-hand side of (2.109) should be $c_1|\hat{u} - I_h\hat{u}|^2_{H^2(\Delta^*)}$.

Assumption 2.64 in general can only be verified for constant boundary data $g$; $H^2$-regularity results usually include an extra term on the right-hand side to account for $g \neq 0$ – see (Renardy and Rogers, 2004, Theorem 8.53) or (McLean, 2000, Theorem 4.10).

**p83** In Exercise 2.4(a), the jump (2.114) is taken across $x = y$ and is better written

$$\big[G'(x, y)\big]_{x=y_-}^{x=y_+} = \frac{-1}{p(y)},$$

that is, the limit of $G'(y + \epsilon, y) - G'(y - \epsilon, y)$ as $\epsilon$ approaches 0 from above. As printed, $G'$ denotes $\partial G/\partial x$ but the two limits are taken in the second argument, so the sentence does not describe the quantity being computed: the derivation integrates the equation over $x \in [y - \epsilon, y + \epsilon]$ at fixed $y$. Symmetry of $G$ makes the two expressions agree numerically, so nothing that follows is affected.

**p84** In Exercise 2.11(b), the inequality holds as stated but does not follow by the method of part (a): Exercise 1.14(a) applies to functions vanishing at an endpoint, and $e'$ need not vanish at either endpoint of $(0, h)$. Since $e(0) = e(h) = 0$, Rolle's theorem gives $\xi \in (0, h)$ with $e'(\xi) = 0$; writing $e'(y) = \int_\xi^y e''(s)\,\mathrm{d}s$ and applying the Cauchy–Schwarz inequality then gives the bound with constant 1.

Chapter 3. **p97** In Example 3.13, the Fourier transform of $u_x$ should be $\widehat{u_x}(\lambda) = \mathrm{i}\lambda\hat{u}$ and not $-\mathrm{i}\lambda\hat{u}$. This is the identity derived in Example 1.105, from which it is quoted. Equation (3.25) is unaffected, since the symbol is squared.

**p105** In the proof of Lemma 3.30, the displayed bound should read

$$\int_D \tilde{f}(u(\boldsymbol{x}))^2\,d\boldsymbol{x} \le (L')^2 \int_D \big(1 + |u(\boldsymbol{x})|\big)^2\,d\boldsymbol{x},$$

with the modulus of $u$ and with $L' = \max\{L, |\tilde{f}(0)|\}$ in place of the Lipschitz constant $L$. As printed, with $(1 + u(\boldsymbol{x}))^2$, the inequality fails for negative $u$: for $\tilde{f}(r) = r$ (Lipschitz with $L = 1$) and $u \equiv -1$ the left-hand side is $|D|$ and the right-hand side is 0.

**p107** In (3.46) the remainder term should be $\varepsilon\, r_j(t)$, and correspondingly the last term of (3.47) should be $\varepsilon\,\boldsymbol{r}(t)$. The remainder $r_j$ is defined in (3.45) as that of the difference approximation of $u_{xx}$, which is multiplied by $\varepsilon$ on substitution into (3.44). The $O(h^2)$ conclusion is unchanged.

**p115** In (3.71) and (3.72), the factor should be $(1 + \Delta t\, \lambda_j)^{-1}$ and not $(1 + \Delta t\, \varepsilon\lambda_j)^{-1}$. The eigenvalues are defined in Example 3.39 as $\lambda_j = \varepsilon j^2$, so the $\varepsilon$ is already included; as printed the factor is $(1+\Delta t\, \varepsilon^2 j^2)^{-1}$. Compare the general scheme (3.68), the system (3.70), and Algorithm 3.5, all of which carry a single $\varepsilon$.

**p116** `meshgrid` is misused in Example 3.40 and Algorithm 3.6. See `exa_3.40.m`. In consequence Figure 3.5(b) was computed not from the stated initial data $u_0 = \sin(x_1)\cos(\pi x_2/8)$ but from $\sin(x_2)\cos(\pi x_1/8)$, which jumps by 1.78 and 0.29 across the two periodic boundaries and so cannot have been intended. The online code now uses the stated data and therefore no longer reproduces the printed figure.

**p116** In Example 3.40, the eigenvalues are likewise defined to include the diffusion coefficient, $\lambda_{j_1,j_2} = \varepsilon((2\pi j_1/a_1)^2 + (2\pi j_2/a_2)^2)$. The right-hand side of (3.73) should therefore be $-\lambda_{j_1,j_2}\widehat{u}_{j_1,j_2} + \widehat{f}_{j_1,j_2}$, and the entries of $M$ should be $m_{i,j} = \lambda_{j_1,j_2}$, in both cases without the additional $\varepsilon$. Algorithm 3.6 uses a single $\varepsilon$.

**p119** In the caption of Algorithm 3.7, the spacing of the returned grid should be $h_{\mathrm{ref}} = a/J_{\mathrm{ref}}$ and not $1/J_{\mathrm{ref}}$, the domain being $D = (0, a)$.

**p127** In the proof of Lemma 3.51, the second application of Young's inequality should read

$$2s\left\|\frac{d\rho}{ds}\right\| \|e(s)\| \le s^2\left\|\frac{d\rho}{ds}\right\|^2 + \|e(s)\|^2,$$

the right-hand side carrying the same arguments as the left; as printed it reads $s^2\|\rho(t)\|^2 + \|e(t)\|^2$. The display that follows carries the correct terms and the conclusion is unaffected.

**p129** In Theorem 3.54, the quantifier in (3.98) should read $\forall u_0 \in \mathcal{D}(A^{-\alpha})$ and not $\forall u_0 \in \mathcal{D}(A^{\alpha})$. The bound is stated in terms of $\|u_0\|_{-\alpha}$ and follows by combining (3.94) and (3.96), each stated for $u_0 \in \mathcal{D}(A^{-\alpha})$. The distinction matters: as noted on p128, $\mathcal{D}(A^{-\alpha})$ admits data that need not belong to $H$, and it is that case which is required for space–time white noise forcing later in the book.

**p132** Following the comment on p487 below, the last line of the proof should be

$$\|u(t_n) - \tilde{u}_n\| \le E\ \exp(Ln\Delta t).$$

**p132** In the Notes, the mapping $t \mapsto S(t)u$ of an analytic semigroup is analytic on $(0,\infty)$, and not on $[0,\infty)$, for $u \in X$. Analyticity at $t = 0$ would require $u \in \mathcal{D}(A^n)$ for every $n$; for general $u \in X$ the mapping is only strongly continuous there.

**p134** Exercise 3.4 requires $b > 0$. For $b < 0$ the stated conclusion is false: take $a = 0$, $b = -1$, $z_0 = 1$, and $z(t) = 1/10$ for $t \in [0, 9]$, $z(t) = \mathrm{e}^{-(t-9)}/10$ for $t \ge 9$. Then $z$ satisfies (3.102) for all $t \ge 0$, but $z(5) = 1/10 > \mathrm{e}^{-5}$, whereas the claimed bound is $\mathrm{e}^{bt}z_0 + (a/b)(\mathrm{e}^{bt} - 1) = \mathrm{e}^{-5}$.

**p135** In Exercise 3.9, the step sizes should be related by $\Delta t_0 = \kappa\,\Delta t_1$, so that $\boldsymbol{u}_{N_1}$ is the finer approximation. With $\Delta t_1 = \kappa\,\Delta t_0$ as printed, eliminating $C_1$ gives

$$\frac{\kappa\boldsymbol{u}_{N_1} - \boldsymbol{u}_{N_0}}{\kappa - 1} = \boldsymbol{u}(T) - C_1(\kappa + 1)\Delta t_0 + O(\Delta t_0^2),$$

so the $O(\Delta t_0)$ term does not cancel. The corrected relation also matches Exercise 3.10, where $\Delta t_1 = 0.01$ and $\Delta t_0 = 0.1$.

**p135** In Exercise 3.11, $H_0^1(D) \times L^2(D)$ is the space in which the first-order system evolves, not the domain of the generator; the phrase should read "in the space $H = H_0^1(D) \times L^2(D)$". The domain is the smaller set $\mathcal{D}(A) = (H^2(D) \cap H_0^1(D)) \times H_0^1(D)$.

**p136** In Exercise 3.20, the second inequality requires that $B$ commute with $A$ (for instance, that $B$ be a function of $A$); it is false for general $B \in \mathcal{L}(H)$. Take $H = \mathbb{R}^2$, $A = \mathrm{diag}(1, 4)$, $\alpha = 1/2$, $B = \begin{pmatrix}1 & 1\\ 0 & 0\end{pmatrix}$ and $u = [1, -1/4]^{\mathsf{T}}$. Then $BAu = 0$, so the right-hand side is 0, while $BA^{1/2}u = [1/2, 0]^{\mathsf{T}}$ gives $\|BA^{1/2}u\| = 1/2$.

Chapter 4. **p156** In Example 4.51 (conditional Gaussian), the normalising constant of the pdf of $\boldsymbol{X}$ should read $1/\sqrt{(2\pi)^{n+m}\det Q}$, and each exponent should carry a factor 1/2, in accordance with (4.13); the first display should read

$$p_{\boldsymbol{X}}(\boldsymbol{x}_1, \boldsymbol{x}_2) = \frac{1}{\sqrt{(2\pi)^{n+m}\det Q}}\,\mathrm{e}^{-(\boldsymbol{x}-\boldsymbol{\mu})^{\mathsf{T}}Q^{-1}(\boldsymbol{x}-\boldsymbol{\mu})/2},$$

and similarly for the two displays that follow. As printed, the final display makes the conditional density of $\boldsymbol{X}_2$ given $\boldsymbol{X}_1 = \boldsymbol{y}$ proportional to $\mathrm{e}^{-(\boldsymbol{x}_2-\widehat{\boldsymbol{\mu}}_2)^\top \overline{Q}_{22}^{-1}(\boldsymbol{x}_2-\widehat{\boldsymbol{\mu}}_2)}$, which is the density of $\mathrm{N}(\widehat{\boldsymbol{\mu}}_2, \overline{Q}_{22}/2)$ and not $\mathrm{N}(\widehat{\boldsymbol{\mu}}_2, \overline{Q}_{22})$ as concluded.

**p159** Nensen's inequality → Jensen's inequality (in proof of Theorem 4.58(iii)).

**p159** In the proof of Lemma 4.57 (Borel–Cantelli), the last sentence should read: for $\omega \notin E$, $|X_n(\omega)| > 1$ for only finitely many $n$, so $|X_n(\omega)| \le 1$ for all $n \ge n^*(\omega)$; since $\mathbb{P}(E) = 0$, (4.22) holds. The assertion $\mathbb{P}(n^* = 1) = 0$ does not follow and is false in general: if $X_n = 0$ for all $n$, then $\sum_{n=1}^\infty \mathbb{P}(F_n) = 0 < \infty$ and $n^* = 1$ everywhere, so $\mathbb{P}(n^* = 1) = 1$, while (4.22) holds.

**p168** In the caption of Algorithm 4.7, `M` is the number of trials required to obtain `X`, and not the number of rejections: the counter is incremented on the accepted draw as well, so the number of rejections is `M-1`.

**p174** In the first display of the Monte Carlo error analysis, $\overline{X}_N$ should read $\overline{X}_M$, the sample average being taken over the $M$ Monte Carlo samples ($N$ is the number of time steps).

**p177** In Algorithm 4.10, the reported confidence radius `sig95` does not account for the correlation between antithetic pairs. The call `monte(u(:,1))` treats the $2M$ pooled values as independent and returns $2\,\mathrm{std}/\sqrt{2M}$, which is the confidence radius for $\overline{X}_{2M}$. For the antithetic estimator $(\overline{X}_M + \overline{X}_M^{\mathrm{a}})/2$ it should be computed from the pair averages $Y_j = \big(\phi(\boldsymbol{u}_{N,j}) + \phi(\boldsymbol{u}_{N,j}^{\mathrm{a}})\big)/2$, as $2\,\mathrm{std}(Y)/\sqrt{M}$. The reported point estimate is correct.

**p178** In Exercise 4.5, the moments should read $\mathbb{E}\big[|X|^k\big] = \infty$ for $k \ge 1$. For $X \sim \mathrm{Cauchy}(0,1)$ and $k$ odd, $\mathbb{E}[X^k]$ is undefined rather than infinite, since $\mathbb{E}\big[(X^k)^+\big] = \mathbb{E}\big[(X^k)^-\big] = \infty$.

**p179** Bayes' theorem is due to the Reverend Thomas Bayes and the apostrophe is written after and not, as in Exercise 4.11, before the s. In the same exercise, $p_{X,Y}$ is incorrectly defined and it should be

$$\mathbb{P}(X = x_k, Y = y_j) = P_{X,Y}(k,j)$$

(interchange $j$ and $k$).

**p180** In Exercise 4.18, the comparison with "performing a few scalar multiplications" depends on how the multiplication is timed and is best dropped; the comparison with a trigonometric function is robust. Measured against the marginal cost of vectorised arithmetic on current hardware, a uniform sample costs of the order of 30 multiplications and a Gaussian sample of the order of 85. Only when the multiplication is timed as a whole array operation, with allocation included, is the cost of sampling "a few" multiplications.

Chapter 5. **p184** In the proof of Lemma 5.8, the increment should read

$$X_{\lfloor t_{i+1}N\rfloor} - X_{\lfloor t_i N\rfloor} = \sum_{j=\lfloor t_i N\rfloor+1}^{\lfloor t_{i+1}N\rfloor} \xi_{j-1},$$

to match the convention $X_n = \sum_{j=1}^n \xi_{j-1}$ used in the proof of Lemma 5.5.

**p188** In (5.12), the exponent should read $\mathrm{e}^{-(x-x_j)^2/2t_n}$, with a minus sign, as in (5.13).

**p200** In Example 5.26, the covariance function should read $C(s,t) = \big(|t|^{2H} + |s|^{2H} - |t-s|^{2H}\big)/2$, matching the definition of fBm in §5.3.2 and the factor `0.5` used in Algorithm 5.3 (`fbm.m`). The stated conclusion that $C_N$ is singular is unaffected, since $C(0,t) = 0$ for either version.

**p201** In the display bounding $|\mathbb{E}[\phi(X)] - \mathbb{E}[\phi(\widehat{X})]|$, the Frobenius norm should appear to the first power, as in Theorem 4.63, so that the right-hand side reads

$$K\,\|\phi\|_{L^2(\mathbb{R}^N)}\|C_n - C_N\|_F = K\,\|\phi\|_{L^2(\mathbb{R}^N)}\Big(\sum_{j=n+1}^N \nu_j^2\Big)^{1/2}.$$

The estimate holds for $\phi$ bounded and continuous, rather than for all $\phi \in L^2(\mathbb{R}^N)$, because $C_n$ is singular. It follows by extending Theorem 4.63 to positive-semi-definite $C_Y$ under this additional assumption on $\phi$: regularise $C_n$ by $C_n + \delta I$ with $\delta \downarrow 0$, apply Theorem 4.63 (whose constant depends only on $C_N$ and $\|C_n - C_N\|_F$), and pass to the limit using $\widehat{\boldsymbol{X}} + \delta^{1/2}\boldsymbol{Z} \Rightarrow \widehat{\boldsymbol{X}}$ in distribution.

**p203** The statement "If the process is Gaussian" in Theorem 5.28 admits two readings. In the sense of Definition 4.38 ($H$-valued Gaussian, with $H = L^2(\mathcal{T})$), $X$ Gaussian means $\langle X, \phi \rangle$ is a real-valued Gaussian random variable for every $\phi \in H$. Since $\xi_j = \langle X - \mu, \phi_j \rangle_{L^2(\mathcal{T})} / \sqrt{\nu_j}$ is of this form, and so is any linear combination $\sum_j \theta_j \xi_j = \langle X - \mu, \sum_j \theta_j \phi_j / \sqrt{\nu_j} \rangle$, the $\xi_j$ are then immediately (jointly) Gaussian.

If instead "Gaussian" is read in the sense of Definition 5.10 (Gaussian finite-dimensional distributions), the claim needs justification, since $\gamma_j = \langle X - \mu, \phi_j \rangle_{L^2(\mathcal{T})}$ is an integral of $X$. It suffices that every linear combination $S = \sum_{j=1}^n \theta_j \gamma_j = \langle X - \mu, \psi \rangle_{L^2(\mathcal{T})}$, $\psi = \sum_{j=1}^n \theta_j \phi_j$, is univariate Gaussian. If $X$ is mean-square continuous (equivalently, its covariance is continuous, the setting of Theorem 5.29), the Riemann sums $S_m = \sum_i (X(t_i) - \mu(t_i))\psi(t_i)\,\Delta t_i$ converge to $S$ in $L^2(\Omega)$; each is a finite linear combination of the $X(t_i)$, hence Gaussian by Definition 5.10, with characteristic function $\mathbb{E}[\mathrm{e}^{\mathrm{i}rS_m}] = \exp(\mathrm{i}r\,\mathbb{E}[S_m] - \frac{1}{2}r^2\mathrm{Var}(S_m))$, and letting $m \to \infty$ (the means and variances converge) gives $\mathbb{E}[\mathrm{e}^{\mathrm{i}rS}] = \exp(\mathrm{i}r\,\mathbb{E}[S] - \frac{1}{2}r^2\mathrm{Var}(S))$, so $S$ is Gaussian. For a merely square-integrable covariance, a jointly measurable process satisfying Definition 5.10 induces a Gaussian measure on its path space, of which each $\gamma_j = \langle X - \mu, \phi_j \rangle$ is a measurable linear functional, hence Gaussian; that is, $X$ is an $H$-valued Gaussian in the sense of Definition 4.38 (see, e.g., Bogachev, *Gaussian Measures*, 1998, Proposition 2.3.9), which returns to the first case.

**p204** The equality in (5.39) follows from the uniform-in-time Mercer convergence in Theorem 5.29.

**p207** In Definition 5.31, the identity should read

$$\|X(t+h) - X(t)\|_{L^2(\Omega)} = \mathbb{E}\big[|X(t+h) - X(t)|^2\big]^{1/2} \to 0 \quad \text{as } h \to 0,$$

with the exponent $1/2$, as in Definition 5.33.

**p210** In the proof of Proposition 5.37, the display following the appeal to Definition 5.33 should use the difference quotient over $[t_j, t_{j+1}]$, matching the identity established at the start of the proof:

$$\frac{X(t) - X(t_j)}{t - t_j} - \frac{X(t_{j+1}) - X(t_j)}{t_{j+1} - t_j} \to \frac{dX(t)}{dt} - \frac{dX(t)}{dt} = 0 \quad \text{in } L^2(\Omega).$$

**p211** In Example 5.41, the sentence should read: hence if $n > 1/(2H)$, Theorem 5.40 applies with $p = 2n$ and $r = 2nH - 1 > 0$. As printed, with $p = \lceil 1/H \rceil$ and $r = pH - 1$, one has $r = 0$ whenever $1/H$ is an integer — in particular $r = 0$ for $H = 1/2$, the case invoked in Corollary 5.42 — whereas (5.50) requires $r > 0$. Taking $p = 2n$ also matches the even moments supplied by (5.51).

**p213** In the proof of Theorem 5.40, the second branch of the definition of $Y$ should read $(1-t)X(0) + t\,X(1)$ and not $X(0) + t\,X(1)$. With the latter, $Y(1) = X(0) + X(1) \neq X(1)$ unless $X(0) = 0$, contrary to the note that $Y(t) = X(t)$ for $t = 0, 1$.

**p215** In Exercise 5.9, the process should read $B(t) = b\,t/T + W(t) - \frac{t}{T}W(T)$. As printed, the centred part of $B(t) = b\,t/T + (1 - t/T)W(t)$ has covariance $(1 - s/T)(1 - t/T)\min\{s,t\} = s(T-s)(T-t)/T^2$ for $s \le t$, and not the Brownian bridge covariance $\min\{s,t\} - st/T = s(T-t)/T$ of Lemma 5.22; the two agree only at $s = 0$.

**p215** In Exercise 5.10, fBm $B^H(t)$ has long-range dependence for $H > 1/2$, not $H < 1/2$. With $\Delta(n) = B^H(n) - B^H(n-1)$,

$$\sum_{n=1}^{K} \mathbb{E}\big[\Delta(1)\Delta(n)\big] = \frac{1}{2} + \frac{1}{2}\big(K^{2H} - (K-1)^{2H}\big),$$

which converges to $1/2$ as $K \to \infty$ for $H < 1/2$ and diverges for $H > 1/2$.

Chapter 6. **p217** In the chapter introduction, the stationary process obtained as $t \to \infty$ from

$$\frac{du}{dt} = -\lambda u + \sqrt{2}\,\zeta(t)$$

has covariance $C(s,t) = \frac{1}{\lambda}\mathrm{e}^{-\lambda|s-t|}$. The stated covariance $C(s,t) = \mathrm{e}^{-|s-t|}$ holds only for $\lambda = 1$.

**p221** In Example 6.8, the special case $q = 1/2$ of the Whittle–Matérn covariance should read $c_{1/2}(t) = \mathrm{e}^{-|t|}$, in agreement with (6.5), which defines $c_q$ in terms of $|t|$.

**p221** The identity (6.7) requires $\nu \neq 0$. For a stationary process of mean $\mu$, the constant part contributes $\mu^2\big|\int_0^T \mathrm{e}^{-\mathrm{i}\nu t}\,dt\big|^2/(2\pi T)$, which is bounded by $2\mu^2/(\pi T\nu^2)$ and so vanishes as $T \to \infty$ whenever $\nu \neq 0$; at $\nu = 0$ it equals $\mu^2 T/2\pi$ and diverges. Estimates of the spectral density from data are correspondingly useless at $\nu = 0$ unless the mean is subtracted first.

**p221** In Example 6.9, the approximation of the spectral density should read

$$f_T(\nu_k) \approx \frac{T}{2\pi} U_k^2, \qquad \text{for } k = -J/2+1, \dots, J/2,$$

with the factor $T$. The identity quoted immediately above is $f_T(\nu_k) = (T/2\pi)u_k^2$, and $U_k$ is the trapezium rule approximation to $u_k$ with the same normalisation; Algorithm 6.1 (`spectral_density.m`) correctly computes $T|U_k|^2/2\pi$.

**p233** In Example 6.30, the factor 2 in front of the expectation should be deleted, so that the covariance reads

$$C_Z(s,t) = \mathbb{E}\big[Z(s)\overline{Z(t)}\big] = 2\int_{\mathbb{R}} \mathrm{e}^{\mathrm{i}(s-t)\nu} \frac{\ell}{\sqrt{4\pi}} \mathrm{e}^{-\nu^2\ell^2/4}\, d\nu = 2c(s-t),$$

as in Example 6.29 and as required by the definition of $C_Z$ on p232. The final value $2c(s-t)$ is correct.

**p235** In Algorithm 6.3 (`quad_sinc.m`), the normalisation of the increments $\Delta W_j$ by the variance $f(\nu_j)\,\Delta\nu_j$ is incomplete. The nodes $\nu_j$ span $[-R, R]$ in $J-1$ steps, so `nustep` should be $\Delta\nu = 2R/(J-1)$ (line 2), and then $f(\nu_j)\,\Delta\nu_j = (\ell/2\pi)\cdot 2(\pi/\ell)/(J-1) = 1/(J-1)$. The final scaling (last line) should therefore read `Z=Z*sqrt(1/(J-1))` in place of `Z=Z*sqrt(ell/(2*pi))`, the factor $\ell/2\pi$ being subsumed.

**p239** The display (6.34) is missing the term $\sum_{k=0}^{N-1} e^{i2\pi Nk/N}$, which is the FFT part.

**p240** In the proof of Corollary 6.37, the display following the citation of Lemma 6.36 should read

$$|c(s-t) - \widetilde{c}_R(s-t)| \le K_1\Big(\frac{1}{M^2}\frac{1}{N^{2q}} + \frac{N}{M^4} + \frac{1}{N^{2q}}\Big), \qquad \forall\, s,t \in [0,T],$$

with $c$ in place of $c_Z$: Lemma 6.36 bounds $|c(t) - \widetilde{c}_R(t)|$, and as printed the bound is false, since $|c_Z - \widetilde{c}_R| = |2c - \widetilde{c}_R| \to |c|$ as $\widetilde{c}_R \to c$. In the same proof, the linear interpolant $I(t)$ is that of $\Re\widetilde{Z}_R(t)$, which has covariance $\widetilde{c}_R(t)$ and not $\widetilde{c}_R(t)/2$, because $\widetilde{Z}_R(t)$ has covariance $2\widetilde{c}_R(t)$; and display (6.36) should read

$$\max_{j,k=0,\dots,P-1} \big|c(s_j - s_k) - C_I(s_j, s_k)\big| \le K_1\frac{N}{M^4} + \frac{K_1+K_2}{N^2},$$

consistent with the entries $c_Z(s_j - s_k)/2 = c(s_j - s_k)$ of $C_{\text{true}}$. The rate (6.35) is unaffected.

**p241** At the start of §6.5, the vector of samples should read

$$\boldsymbol{X} = \big(X(t_0), X(t_1), \dots, X(t_{N-1})\big)^{\mathsf{T}}, \qquad t_n = n\Delta t.$$

**p256** In Exercise 6.18, the second identity should read

$$d_k = c_0 + c_P(-1)^{k-1} + 2\sum_{j=1}^{P-1} c_j \cos\Big(\frac{\pi j(k-1)}{P}\Big), \qquad k = 1, \dots, 2P;$$

that is, $c_{P/2}$ should be $c_P$ and the upper limit $P/2-1$ should be $P-1$. For a $2P \times 2P$ circulant, $\omega = \mathrm{e}^{\pi\mathrm{i}/P}$, so the self-paired term of the first sum is $j = P$, with $\omega^{P(k-1)} = (-1)^{k-1}$, and the conjugate pairs $\pm j$ run over $j = 1, \dots, P-1$; note $\omega^{P/2} = \mathrm{i}$, so $c_{P/2}(-1)^{k-1}$ cannot arise. The stated range $k = 1, \dots, P$ gives only $P$ of the $2P$ eigenvalues.

**p256** Exercise 6.21(b) is ambiguous, and under its literal reading the answer is negative. Fixing $\Delta t = 1/(N-1)$ ties the sample points to $[0,1]$, so increasing $N$ refines the grid rather than lengthening the interval, and refining never helps: for $\ell = 1$, the minimal extension has $\rho(D_-) = 3.83, 7.70, 15.4, 30.9$ at $N = 100, 200, 400, 800$. Only $\ell = 0.1$, for which $[0,1]$ already spans ten correlation lengths, gives a non-negative definite $\widetilde{C}$ at every $N$. The intended reading is to pad, that is, to extend the sampled interval at fixed spacing $\Delta t$; the minimal extension then becomes non-negative definite once the interval covers about five correlation lengths.

Chapter 7. **p259** In Example 7.7, the pdf of $u(\boldsymbol{x}) = W_1(x_1)W_2(x_2)$ should read

$$p(u) = \frac{1}{\pi\sqrt{x_1x_2}} K_0\Big(\frac{|u|}{\sqrt{x_1x_2}}\Big),$$

with $\sqrt{x_1x_2}$ in place of $x_1x_2$ in both positions. As printed, the density has variance $(x_1x_2)^2$, whereas the example's own covariance computation gives $\mathbb{E}[u(\boldsymbol{x})^2] = x_1x_2$.

**p262** In Example 7.12, the spectral density of the separable exponential covariance should read

$$f(\boldsymbol{\nu}) = \prod_{i=1}^{d} \frac{\ell_i}{\pi\,(1+\ell_i^2\nu_i^2)},$$

as computed for each factor in Example 6.6, equation (6.4). At $\boldsymbol{\nu} = \mathbf{0}$ the printed formula gives $\prod_i 1/(\pi\ell_i)$, whereas $f(\mathbf{0}) = (2\pi)^{-d}\int_{\mathbb{R}^d} c(\boldsymbol{x})\,d\boldsymbol{x} = \prod_i \ell_i/\pi$; the two agree only when every $\ell_i = 1$.

**p274** In Example 7.36, the definition of $\widetilde{C}_i$ should read $i = 0, 1$ and not $i = 1, 2$.

**p278** In Algorithm 7.3 (`gaussA_exp.m`), the cross term should read `+2*x1*x2*a12`, so that the code evaluates $c(\boldsymbol{x}) = \mathrm{e}^{-\boldsymbol{x}^{\mathsf{T}}A\boldsymbol{x}}$ for the matrix $A$ of (7.32). As printed, the code returns $\mathrm{e}^{-\boldsymbol{x}^{\mathsf{T}}A\boldsymbol{x}}$ with $A$ having off-diagonal entries $-a_{12}$: the matrix $C_{\mathrm{red}}$ displayed in Example 7.40 is the one obtained for $a_{12} = -0.5$, its $(1,1)$ entry being $c(-1,-1) = \mathrm{e}^{-1} = 0.3679$ rather than $\mathrm{e}^{-3} = 0.0498$. With the corrected code, the first and third columns of the displayed $C_{\mathrm{red}}$ are interchanged.

**p279** The matrix V in the display at the top of the page has index typos. The first two entries of the second column should contain the indices $2n_1+1$ and $2n_1+2$ instead of $n_2$.

**p285, p286, p289** In Example 7.45, the turning bands operator applied to the Bessel covariance should read

$$T_d\big(\Gamma(d/2)\,J_p(t)/(t/2)^p\big) = \cos(t), \qquad p = (d-2)/2,$$

so that $c_X(t) = \cos(t)$ and the factors $1/\sqrt{2}$ should be deleted from (7.34) and (7.35), giving $X(t) = \cos(t)\xi_1 + \sin(t)\xi_2$; correspondingly `sqrt(1/2)` should be deleted from Algorithm 7.7 (`turn_band_simple.m`) and Algorithm 7.9 (`turn_band_simple2.m`). By Definition 7.43 with $F^0(s) = \delta(s-1)$, $T_d c^0(t) = \int_0^\infty \cos(st)\,dF^0(s) = \cos(t)$. With $c_X = \frac{1}{2}\cos$, Lemma 7.46 gives the turning bands field the covariance $\frac{1}{2\pi}\int_0^{2\pi}\frac{1}{2}\cos(r\cos\phi)\,d\phi = \frac{1}{2}J_0(r)$ when $d=2$, half the Bessel covariance $c^0(r) = J_0(r)$ of Example 7.18, contradicting Proposition 7.47.

**p286** In the proof of Proposition 7.47, the displayed Bessel identity should read

$$\frac{J_p(r)}{(r/2)^p} = \frac{2}{\pi^{1/2}\Gamma((d-1)/2)}\int_0^{\pi/2}\cos(r\cos\theta)\sin^{d-2}\theta\,d\theta, \qquad p = \frac{d-2}{2};$$

that is, a factor $r^{(d-2)/2}$ is missing from the printed coefficient $2^{2-d/2}/(\pi^{1/2}\Gamma((d-1)/2))$ (it is printed correctly in Appendix A.3).

**p291** In Algorithm 7.10 (`turn_band_wm.m`), `f` should be an even function of `s` (missing modulus).

**p291** In the proof of Corollary 7.50, $|c^0 - \tilde{c}_M| = |c^0 - c_M| + |c_M - \tilde{c}_M|$ should read $|c^0 - \tilde{c}_M| \le |c^0 - c_M| + |c_M - \tilde{c}_M|$.

**p305** In Lemma 7.67, the quantity $M_0(u)$ is infinite as stated and cannot be used for proving Theorem 7.68. We show here how to prove Theorem 7.68 using the Garsia–Rodemich–Rumsey (GRR) inequality. We state GRR on a bounded domain $D \subset \mathbb{R}^d$, rather than only on $D = (0,1)$, so that the argument covers Theorem 7.68 as stated. For an alternative approach to Theorem 7.68, see M. Talagrand, ‘Upper and Lower Bounds for Stochastic Processes’, Springer, 2021 (Section 2.5).

Let $\Psi(t) = \exp(t^2)$ and $p(t) = t^q$. Throughout, $D \subset \mathbb{R}^d$ is a bounded domain whose boundary is piecewise smooth, as is assumed throughout the book. All that is used of this is the resulting *measure-density* property: writing $B(\boldsymbol{x}, r)$ for the ball of radius $r$ centred at $\boldsymbol{x}$, there is a $c_D > 0$ with

$$\mathrm{Leb}(D \cap B(\boldsymbol{x}, r)) \ge c_D\, r^d, \qquad \boldsymbol{x} \in \overline{D}, \quad 0 < r \le \operatorname{diam} D,$$

which prevents $D$ from pinching. After rescaling we may, and do, assume $\operatorname{diam} D \le 1$.

(Note: GRR applies to a wider class of $\Psi$ and $p$, which we do not describe here — see Garsia, Rodemich, and Rumsey (1971) for details. The multidimensional form stated here is due to Garsia, ‘Continuity properties of Gaussian processes with multidimensional time parameter’, Berkeley Symp. on Math. Statist. and Prob., 1972; for $d = 1$ and $D = (0,1)$ one may take $C_d = 8$ and $c_d = 4$.)

**Lemma** (Garsia–Rodemich–Rumsey)**.** *Let $f$ be a continuous function on $\overline{D}$ and suppose that*

$$M_0(f) = \int_D\int_D \Psi\left(\frac{|f(\boldsymbol{x}) - f(\boldsymbol{y})|}{p(\|\boldsymbol{x}-\boldsymbol{y}\|_2)}\right) d\boldsymbol{x}\,d\boldsymbol{y} < \infty.$$

*Then, for constants $C_d, c_d > 0$ depending only on $d$ and $c_D$, we have, for all $\boldsymbol{x}, \boldsymbol{y} \in \overline{D}$,*

$$|f(\boldsymbol{x}) - f(\boldsymbol{y})| \le C_d \int_0^{\|\boldsymbol{x}-\boldsymbol{y}\|_2} \Psi^{-1}\left(\frac{c_d M_0(f)}{r^{2d}}\right) dp(r).$$

The power $r^{2d}$, in place of the $r^2$ of the one-dimensional statement, arises because the estimate is localised at scale $r$ to a pair of sets each of volume comparable with $r^d$; the factor $C_d$ is the 8 of the classical one-dimensional inequality.

**Lemma** (Lemma 7.67 revised)**.** *Let $D \subset \mathbb{R}^d$ be as above, with* $\operatorname{diam} D \le 1$. *Fix $q > \epsilon > 0$. For a constant $C_\epsilon$ depending only on $\epsilon$, $q$, $d$ and $D$, we have, for any $u \in C(\overline{D})$,*

$$|u(\boldsymbol{x}) - u(\boldsymbol{y})| \le C_\epsilon \sqrt{\log(e + M_0(u))}\, \|\boldsymbol{x} - \boldsymbol{y}\|_2^{q-\epsilon}, \qquad \boldsymbol{x}, \boldsymbol{y} \in D, \tag{1}$$

*where we assume*

$$M_0(u) := \int_D \int_D \exp\left(\frac{|u(\boldsymbol{x}) - u(\boldsymbol{y})|^2}{\|\boldsymbol{x} - \boldsymbol{y}\|_2^{2q}}\right) d\boldsymbol{x}\, d\boldsymbol{y} < \infty. \tag{2}$$

*Proof.* With $\Psi(t) = \exp(t^2)$ and $p(t) = t^q$,

$$M_0(u) = \int_D \int_D \Psi\left(\frac{|u(\boldsymbol{x}) - u(\boldsymbol{y})|}{p(\|\boldsymbol{x} - \boldsymbol{y}\|_2)}\right) d\boldsymbol{x}\, d\boldsymbol{y},$$

which is finite by assumption, so GRR applies. Note that $\Psi(t) \ge 1$ for $t \ge 0$, that $\Psi^{-1}(t) = \sqrt{\log t}$ for $t \ge 1$, and that $p'(t) = q\, t^{q-1}$. Put $N := \max\{c_d M_0(u), e\}$, so that $\log N \ge 1$ and, for a constant $\kappa \ge 1$ depending only on $c_d$, $\log N \le \kappa \log(e + M_0(u))$. Since $\Psi^{-1}$ is increasing and $r \le 1$,

$$\begin{aligned}
\sqrt{\log(c_d M_0(u)/r^{2d})} &\le \sqrt{\log N + 2d\, |\log r|} \\
&\le \sqrt{\log N} + \frac{d}{\sqrt{\log N}} |\log r| \\
&\le \sqrt{\log N} + \frac{d}{\epsilon\, e}\, r^{-\epsilon},
\end{aligned}$$

using $\log N \ge 1$, $\sqrt{a + b} \le \sqrt{a} + \frac{b}{2\sqrt{a}}$ for $a, b > 0$, and $|\log r| \le r^{-\epsilon}/\epsilon e$ for $r \in (0, 1)$. Then, from GRR, for any $\boldsymbol{x}, \boldsymbol{y} \in D$,

$$\begin{aligned}
|u(\boldsymbol{x}) - u(\boldsymbol{y})| &\le C_d \int_0^{\|\boldsymbol{x}-\boldsymbol{y}\|_2} \sqrt{\log\left(\frac{c_d M_0(u)}{r^{2d}}\right)}\, q\, r^{q-1}\, dr \\
&\le C_d \sqrt{\log N}\, \|\boldsymbol{x} - \boldsymbol{y}\|_2^q + \frac{C_d\, d}{\epsilon\, e}\, \frac{q}{q - \epsilon}\, \|\boldsymbol{x} - \boldsymbol{y}\|_2^{q-\epsilon}.
\end{aligned}$$

As $\|\boldsymbol{x} - \boldsymbol{y}\|_2 \le \operatorname{diam} D \le 1$, the first term is bounded by $C_d \sqrt{\log N}\, \|\boldsymbol{x} - \boldsymbol{y}\|_2^{q-\epsilon}$, and $\sqrt{\log N} \le \sqrt{\kappa}\sqrt{\log(e + M_0(u))}$, while $\sqrt{\log(e + M_0(u))} \ge 1$ absorbs the second term. This gives (1) for a suitable $C_\epsilon$. □

This version of Lemma 7.67 can be used in the proof of Theorem 7.68.

**Theorem** (Theorem 7.68 revised)**.** *Let $D \subset \mathbb{R}^d$ be a bounded domain and $\{u(\boldsymbol{x}) \colon \boldsymbol{x} \in \overline{D}\}$ be a mean-zero Gaussian random field such that, for some $L, s > 0$,*

$$\mathbb{E}\left[|u(\boldsymbol{x}) - u(\boldsymbol{y})|^2\right] \le L \|\boldsymbol{x} - \boldsymbol{y}\|_2^s, \qquad \forall \boldsymbol{x}, \boldsymbol{y} \in \overline{D}. \tag{3}$$

*For any $p \ge 1$ and any $\epsilon \in (0, s)$, there exists a random variable $K$ such that $e^K \in L^p(\Omega)$ and*

$$|u(\boldsymbol{x}) - u(\boldsymbol{y})| \le K(\omega) \|\boldsymbol{x} - \boldsymbol{y}\|_2^{(s-\epsilon)/2}, \qquad \forall \boldsymbol{x}, \boldsymbol{y} \in \overline{D}, \quad a.s. \tag{4}$$

*Proof.* We can find a truncated Karhunen–Loève expansion $u_J(\boldsymbol{x})$ such that $u_J(\cdot, \omega) \in C(\overline{D})$ for all $\omega \in \Omega$. Fix $p \ge 1$ and let

$$\Delta_J(\boldsymbol{x}, \boldsymbol{y}) := \frac{u_J(\boldsymbol{x}) - u_J(\boldsymbol{y})}{\sqrt{8\, L \|\boldsymbol{x} - \boldsymbol{y}\|_2^s}}, \qquad \boldsymbol{x}, \boldsymbol{y} \in \overline{D}.$$

The random variables in the Karhunen–Loève expansion are *iid* with mean zero and hence $\mathbb{E}\left[\Delta_{J+1} \,\middle|\, \Delta_J\right] = \Delta_J$. Further, the function $t \mapsto \exp(t^2)$ is convex and Jensen's inequality implies that $\mathbb{E}\left[e^{\Delta_{J+1}^2} \,\middle|\, \Delta_J\right] \ge e^{\Delta_J^2}$. That is, $e^{\Delta_J^2}$ is a submartingale and Doob's submartingale inequality gives

$$\mathbb{E}\left[\sup_{1 \le j \le J} e^{2\Delta_j(\boldsymbol{x},\boldsymbol{y})^2}\right] \le 4\mathbb{E}\left[e^{2\Delta_J(\boldsymbol{x},\boldsymbol{y})^2}\right].$$

Note that $\mathbb{E}\big[\Delta_J(\boldsymbol{x},\boldsymbol{y})^2\big] \le L\|\boldsymbol{x}-\boldsymbol{y}\|_2^s/8\,L\|\boldsymbol{x}-\boldsymbol{y}\|_2^s \le 1/8$ for every $\boldsymbol{x},\boldsymbol{y}$. Consequently, $\Delta_J \sim \mathrm{N}(0,\sigma^2)$ where the variance satisfies $\sigma^2 \le 1/8$. As $1/2\sigma^2 - 2 \ge 1/4\sigma^2$,

$$\mathbb{E}\Big[e^{2\Delta_J(\boldsymbol{x},\boldsymbol{y})^2}\Big] = \frac{1}{\sqrt{2\pi\sigma^2}}\int_{\mathbb{R}} e^{2x^2}e^{-x^2/2\sigma^2}\,dx \le \frac{1}{\sqrt{2\pi\sigma^2}}\int_{\mathbb{R}} e^{-x^2/4\sigma^2}\,dx = \sqrt{2}.$$

We conclude that

$$\mathbb{E}\left[\sup_{1\le j\le J} e^{2\Delta_j(\boldsymbol{x},\boldsymbol{y})^2}\right] \le 4\sqrt{2}, \qquad \forall \boldsymbol{x},\boldsymbol{y}\in\overline{D}.$$

Let

$$M_1(u) := \int_D\int_D \sup_{J\in\mathbb{N}} e^{2\Delta_J(\boldsymbol{x},\boldsymbol{y})^2}\,d\boldsymbol{x}\,d\boldsymbol{y},$$

so that, by monotone convergence, $\mathbb{E}[M_1] \le 4\sqrt{2}\,\mathrm{Leb}(D)^2 < \infty$ and $M_1 \in L^1(\Omega)$. Now

$$2\Delta_J(\boldsymbol{x},\boldsymbol{y})^2 = \frac{|u_J(\boldsymbol{x})-u_J(\boldsymbol{y})|^2}{4L\,\|\boldsymbol{x}-\boldsymbol{y}\|_2^s} = \frac{|v_J(\boldsymbol{x})-v_J(\boldsymbol{y})|^2}{\|\boldsymbol{x}-\boldsymbol{y}\|_2^{2q}}, \qquad v_J := \frac{u_J}{2\sqrt{L}}, \quad q := \frac{s}{2},$$

so that $M_0(v_J) \le M_1(u)$ for every $J$. Applying Lemma 7.67 (revised) to $v_J$, with $\epsilon/2$ in place of $\epsilon$, and multiplying through by $2\sqrt{L}$,

$$|u_J(\boldsymbol{x},\omega) - u_J(\boldsymbol{y},\omega)| \le K(\omega)\|\boldsymbol{x}-\boldsymbol{y}\|_2^{(s-\epsilon)/2}, \qquad \forall \boldsymbol{x},\boldsymbol{y}\in\overline{D},$$

for $K(\omega) = 2\sqrt{L}\,C_\epsilon\sqrt{\log(e+M_1(u))}$, which does not depend on $J$.

It remains to show $e^K \in L^p(\Omega)$. Put $a := 2\sqrt{L}\,C_\epsilon$. For $x\ge 0$ and $\delta > 0$ we have $\sqrt{x} \le \delta x + 1/4\delta$, since $\left(\sqrt{\delta x} - 1/2\sqrt{\delta}\right)^2 \ge 0$; taking $\delta = 1/pa$ and $x = \log(e+M_1(u))$ gives

$$p\,K = pa\sqrt{\log(e+M_1(u))} \le \log(e+M_1(u)) + \frac{(pa)^2}{4}.$$

Hence $\mathbb{E}\big[e^{pK}\big] \le e^{(pa)^2/4}(e+\mathbb{E}[M_1]) < \infty$, and $e^K \in L^p(\Omega)$.
As $u_J(\boldsymbol{x}) \to u(\boldsymbol{x})$ as $J\to\infty$ almost surely, this gives (4). □

**p297** In Theorem 7.57, $V_d$ should be the volume of the unit ball in $\mathbb{R}^d$, namely $V_d = \pi^{d/2}/\Gamma(d/2+1)$, and not the volume of the unit sphere. The derivation of the theorem given in the Notes uses $V_d$ in exactly this sense, via $\mathrm{Leb}(\{\boldsymbol{v}\in\mathbb{R}^d : \|\boldsymbol{v}\| < R\}) = V_d R^d$.

**p299** In Theorem 7.61, the covariance should be cited as (7.52) and not (7.48): (7.48) is the one-dimensional covariance $\mathrm{e}^{-|x-y|/\ell}$ on $D = [-a,a]$, whereas the theorem concerns the separable exponential covariance with correlation lengths $\ell_1,\ell_2$ on $D = [-a_1,a_1]\times[-a_2,a_2]$.

**p299** Theorem 7.61 should also be stated for $J \ge 2$. As printed it asserts $E_1 \le 0$, since $\log^2 J/J$ vanishes at $J = 1$, whereas $E_1 > 0$ for a non-degenerate field. The proof requires $J \ge \mathrm{e}^2$ in any case, that being the range in which Exercise 7.17 is valid.

**p300** Following (7.57), the vector $\widehat{\boldsymbol{u}}_J$ has distribution $\mathrm{N}(\boldsymbol{\mu},\widehat{C}_J)$ and not $\mathrm{N}(\boldsymbol{0},\widehat{C}_J)$.

**p303** In the display following (7.63), both occurrences of $\varphi_j$ should read $\varphi_k$:

$$\sum_{k=1}^{P} a_k^j \int_D\int_D C(\boldsymbol{x},\boldsymbol{y})\,\varphi_k(\boldsymbol{y})\varphi_i(\boldsymbol{x})\,d\boldsymbol{y}\,d\boldsymbol{x} = \hat{\nu}_j \sum_{k=1}^{P} a_k^j \int_D \varphi_k(\boldsymbol{x})\varphi_i(\boldsymbol{x})\,d\boldsymbol{x}.$$

**p307** In the proof of Corollary 7.70, the exponent $(q-\epsilon)/2$ should read $(s-\epsilon)/2$ at all three occurrences, matching (7.74). Thus $K$ is chosen with $\mathrm{e}^K \in L^q(\Omega)$ for $q = 2\lambda p\,|R|^{(s-\epsilon)/2} \ge 1$, and

$$\mathrm{e}^{\lambda p(u(\boldsymbol{x})-u(\boldsymbol{0}))} \le \mathrm{e}^{\lambda p K(\omega)\|\boldsymbol{x}\|^{(s-\epsilon)/2}} \le \mathrm{e}^{\lambda p K(\omega)|R|^{(s-\epsilon)/2}} = \mathrm{e}^{qK(\omega)/2}.$$

**p310** In Exercise 7.1(1), the covariance should read $\mathbb{E}[W_N(\boldsymbol{x})W_N(\boldsymbol{y})] = \min\{x_1,y_1\}\min\{x_2,y_2\}$, and in Exercise 7.2(b) the distribution should read $\mathrm{N}(0,\,x_1x_2 + y_1y_2 - 2(x_1\wedge y_1)(x_2\wedge y_2))$. In both, the minimum is taken across the two points, as in the Brownian sheet covariance of Example 7.7.

Chapter 8. **p315, p316** In the captions of Figures 8.1(a) and 8.2(b), the horizontal lines mark the 95% confidence interval and not the 90% interval. The interval $\left[-2\sqrt{\sigma^2/2\lambda},\ +2\sqrt{\sigma^2/2\lambda}\right]$ is $\pm 2$ standard deviations of $\mathrm{N}(0,\sigma^2/2\lambda)$ and carries probability 0.954; the 90% interval is $\pm 1.645\sqrt{\sigma^2/2\lambda}$.

**p317** In §8.2, the $\sigma$-algebra should be the natural filtration $\mathcal{F}_s = \sigma(W(u) \colon u \le s)$ and not $\sigma(W(s))$. With $\mathcal{F}_s = \sigma(W(s))$, the identity $\mathbb{E}[W(t) \mid \mathcal{F}_s] = W(s \wedge t)$ fails for $t < s$: it gives $\mathbb{E}[W(t) \mid \sigma(W(s))] = (t/s)W(s)$, not $W(t)$ (the Brownian bridge).

**p322** In Definition 8.12, the left-hand side of (8.18) should be squared,

$$\|I_n(t) - I_m(t)\|^2_{L^2(\Omega)} = \mathbb{E}\left[|I_n(t) - I_m(t)|^2\right] = \int_0^t \mathbb{E}\left[|X_n(s) - X_m(s)|^2\right] ds,$$

as in the Itô isometry (8.17).

**p325** In (8.29), in the proof of Theorem 8.18, the constant should be $L$ and not $L^2$: the linear growth bounds (8.24) of Assumption 8.17 carry the constant $L$. The same replacement is needed in the display from which (8.29) follows. The constant $L^2$ in the contraction estimate later in the proof is correct, as it comes from squaring the Lipschitz bound (8.25).

**p326** In the proof of Lemma 8.19, the limit of $\mathbb{E}[B_N]$ should read

$$\to \frac{1}{2}\int_0^t \phi''(u(s))\, g(u(s))^2\, ds, \qquad \text{as } N \to \infty,$$

with $g^2$ in place of $g$, in agreement with $\mathbb{E}[C_j] = f^2\,\Delta t + g^2$ on the preceding line and with the $g^2$ in the statement (8.31).

**p333** In §8.4, "A simple method for sampling the Lévy areas $A_{ij}$ is considered in Exercise 5.3" should refer to Exercise 8.12, which identifies and approximates the Lévy area $A_{12}$. Exercise 5.3 concerns the Lévy–Ciesielski construction of Brownian motion.

**p335** In Example 8.26, the exponent in the first line of the display for the second moment should be inside the expectation,

$$\mathbb{E}\left[u_n^2\right] = \mathbb{E}\left[\Big(\prod_{j=0}^{n-1}\left(1 + r\,\Delta t + \sigma\,\Delta W_j\right)\Big)^2\right]u_0^2 = \prod_{j=0}^{n-1}\mathbb{E}\left[(1 + r\,\Delta t + \sigma\,\Delta W_j)^2\right]u_0^2.$$

As printed, the left-hand side equals $\left(\mathbb{E}\left[\prod_j(1 + r\,\Delta t + \sigma\,\Delta W_j)\right]\right)^2 u_0^2 = (1 + r\,\Delta t)^{2n}u_0^2$, which is not $\mathbb{E}[u_n^2]$. The conclusion (8.52) is unaffected.

**p340** In the proof of Proposition 8.31(ii), the integrand $\mathbb{E}\left[\|R_G(r; s, \boldsymbol{u}(r))\|_F^2\right]$ should read $\mathbb{E}\left[\|R_G(r; s, \boldsymbol{u}(s))\|_F^2\right]$. The Itô isometry does not change the base point of the integrand.

**p340** In the proof of Proposition 8.31(iii), $\mathbb{E}\left[\|DG(\boldsymbol{u}(s))\boldsymbol{R}_1(s, \boldsymbol{u}(s))\|_F^2\right]$ should read $\mathbb{E}\left[\|DG(\boldsymbol{u}(t_j))\boldsymbol{R}_1(t_j, \boldsymbol{u}(t_j))\|_F^2\right]$, matching the $t_j$-indexed sum and the $R_G$ term alongside it.

**p341** In the proof of Proposition 8.31(iii), the sentence "using (8.43) gives for $r > s$." is incomplete. It should read "where the second equality uses (8.43) with $r > s$ to replace $\boldsymbol{u}(r) - \boldsymbol{u}(s)$."

**p342** In the proof of Theorem 8.32, $\boldsymbol{e}$ is defined only at the grid times, by $\boldsymbol{e}(\hat{t}) = \boldsymbol{u}_{\Delta t}(\hat{t}) - \boldsymbol{u}(\hat{t})$, yet the bounds that follow are written $\mathbb{E}[\|\boldsymbol{e}(s)\|_2^2]$ for all $s$. Define instead $\boldsymbol{e}(t) := \boldsymbol{u}_{\Delta t}(\hat{t}) - \boldsymbol{u}(\hat{t})$ for every $t \in [0, T]$. Then $\boldsymbol{e}$ is piecewise constant and $\mathcal{F}_{\hat{t}}$-measurable, with $\boldsymbol{e}(t_n) = \boldsymbol{u}_n - \boldsymbol{u}(t_n)$, and since $\boldsymbol{D}(s)$ and $M(s)$ depend on the approximation only through $\boldsymbol{u}_{\Delta t}(\hat{s}) - \boldsymbol{u}(\hat{s}) = \boldsymbol{e}(s)$, the displayed bounds hold as printed and Gronwall's inequality applies to $z(t) = \mathbb{E}[\|\boldsymbol{e}(t)\|_2^2]$.

**p342** In the proof of Theorem 8.32, the bound $\mathbb{E}[\|M(s)\|_F^2] \le 2(L + L_2)\,\mathbb{E}[\|\boldsymbol{e}(s)\|_2^2]$ should read

$$\mathbb{E}\left[\|M(s)\|_F^2\right] \le 2\left(L^2 + L_2^2\, m\,(s - \hat{s})\right)\mathbb{E}\left[\|\boldsymbol{e}(s)\|_2^2\right] \le 2\left(L^2 + L_2^2\, m\,\Delta t\right)\mathbb{E}\left[\|\boldsymbol{e}(s)\|_2^2\right].$$

The Lipschitz conditions (8.25) and (8.59) are squared on taking second moments, and the second term carries the factor $\mathbb{E}\left[\|\int_{\hat{s}}^s d\boldsymbol{W}(r)\|_2^2\right] = (s - \hat{s})m$, which factorises out because the increment is independent of $\mathcal{F}_{\hat{s}}$ while $\boldsymbol{e}(s)$ is $\mathcal{F}_{\hat{s}}$-measurable. The existence of $K_3$, and hence the conclusion, is unaffected.

**p351** In the proof of Theorem 8.45, the coefficients of the frozen generator defined after (8.78) should be evaluated at the numerical solution $\boldsymbol{u}_{\Delta t}$ and not at $\boldsymbol{u}$:

$$\widehat{\mathcal{L}}(t)\Phi(t,\boldsymbol{x}) = \boldsymbol{f}(\boldsymbol{u}_{\Delta t}(\hat{t}))^{\mathsf{T}}\nabla\Phi(t,\boldsymbol{x}) + \frac{1}{2}\sum_{k=1}^{m}\boldsymbol{g}_k(\boldsymbol{u}_{\Delta t}(\hat{t}))^{\mathsf{T}}\nabla^2\Phi(t,\boldsymbol{x})\,\boldsymbol{g}_k(\boldsymbol{u}_{\Delta t}(\hat{t})),$$

and $\widehat{\mathcal{L}}^k(t)\Phi(t,\boldsymbol{x}) = \nabla\Phi(t,\boldsymbol{x})^{\mathsf{T}}\boldsymbol{g}_k(\boldsymbol{u}_{\Delta t}(\hat{t}))$. These are the coefficients of (8.77), to which the Itô formula is applied to give (8.78); compare $\Theta_1$ in (8.81), which is evaluated at $\boldsymbol{u}_{\Delta t}(\hat{s})$.

**p351** In the proof of Theorem 8.45, (8.79) should carry an overall minus sign,

$$e = -\mathbb{E}\Big[\int_0^T \frac{\partial}{\partial t}\Phi(T-s,\boldsymbol{u}_{\Delta t}(s)) + \widehat{\mathcal{L}}(s)\Phi(T-s,\boldsymbol{u}_{\Delta t}(s))\,ds\Big],$$

and consequently the display following (8.80) should read

$$e = \mathbb{E}\Big[\int_0^T \big(\mathcal{L}\Phi(T-s,\boldsymbol{u}_{\Delta t}(s)) - \widehat{\mathcal{L}}(s)\Phi(T-s,\boldsymbol{u}_{\Delta t}(s))\big)\,ds\Big].$$

For $d = m = 1$, $\boldsymbol{f} = \boldsymbol{0}$, $g(u) = u$ and $\phi(u) = u^2$, one has $\Phi(t,x) = x^2\mathrm{e}^t$ and $e = u_0^2\big(\mathrm{e}^T - (1+\Delta t)^N\big) > 0$, whereas the display as printed is negative. The theorem is unaffected, since (8.76) bounds $|e|$.

**p353** Below (8.88), "where $\boldsymbol{U}_\ell^{j,\text{fine}}$ and $\boldsymbol{U}_\ell^{j,\text{coarse}}$ are iid samples of $\boldsymbol{U}_\ell$" should read: for each $j$, $\boldsymbol{U}_\ell^{j,\text{fine}}$ is a sample of $\boldsymbol{U}_\ell$ and $\boldsymbol{U}_{\ell-1}^{j,\text{coarse}}$ is a sample of $\boldsymbol{U}_{\ell-1}$, the two being computed from a common Brownian path, and the pairs being independent over $j$. The coarse sample uses the time step $\Delta t_{\ell-1}$ and so is distributed as $\boldsymbol{U}_{\ell-1}$; and the pair is coupled, not independent, as the following sentence states. Were they iid samples of $\boldsymbol{U}_\ell$, then $\mathbb{E}[\mu_\ell] = 0$.

**p361** In Theorem 8.51 (Wong–Zakai), the hypothesis $g(u) \geq g_0 > 0$ should be supplemented by an upper bound $g(u) \leq g_1$, and the two displays in the proof should read

$$|\phi(x) - \phi(y)| = \Big|\int_x^y \frac{1}{g(s)}\,ds\Big| \geq \frac{1}{g_1}|x-y|, \qquad |v(t) - v_J(t)| \leq g_1\,|\phi(v(t)) - \phi(v_J(t))|,$$

with $g_1$ in place of $g_0$. As printed, the hypothesis $g(u) \geq g_0$ gives $1/g \leq 1/g_0$ and hence the reverse inequality $|\phi(x) - \phi(y)| \leq |x-y|/g_0$; a lower bound on $g$ alone does not make $\phi^{-1}$ Lipschitz.

**p368** In Exercise 8.8(a), the limit should read

$$\mathbb{E}[u_n^2] \to \frac{\sigma^2}{2\lambda - \lambda^2\,\Delta t}, \qquad \text{as } n \to \infty,$$

with $2\lambda - \lambda^2\,\Delta t$ in the denominator. The Euler–Maruyama method (8.42) applied to (8.2) is $u_{n+1} = (1-\lambda\,\Delta t)u_n + \sigma\,\Delta W_n$, so that $v_n = \mathbb{E}[u_n^2]$ obeys $v_{n+1} = (1-\lambda\,\Delta t)^2 v_n + \sigma^2\,\Delta t$, with fixed point $\sigma^2\,\Delta t/\big(1-(1-\lambda\,\Delta t)^2\big)$. The limit exists for $\lambda\,\Delta t < 2$ and exceeds the exact value $\sigma^2/2\lambda$.

**p370** In Exercise 8.10(a), $g(u_u)$ should read $g(u_n)$.

**p370** In Exercise 8.10(b), the part is stated for $m = 1$, so there is a single Brownian motion and the increment driving component $k$ should be $\Delta W_n$, not $\Delta W_{k,n}$. The final term of the same display already carries $\Delta W_n^2$.

**p370** In Exercise 8.16(a), "$M = -1$" should read "$A = 1$". No $M$ appears in the semilinear SODE (8.120), in the semi-implicit method (8.121) or in the exact solution (8.122); the linear coefficient is $A$, and $M$ denotes the number of samples elsewhere in the chapter. The sign is fixed by (8.120), where the drift is written $-A\boldsymbol{u} + \boldsymbol{f}(\boldsymbol{u})$: taking $A = 1$ gives the decaying linear part that (8.122) describes, whereas $A = -1$ would give drift $+\boldsymbol{u} + \boldsymbol{f}(\boldsymbol{u})$.

**p371** In (8.121), the trailing $dt$ should be deleted. The step $\Delta t$ already multiplies the drift, and the noise term carries the increment $\Delta\boldsymbol{W}_n$, so the equation reads

$$\boldsymbol{u}_{n+1} = \boldsymbol{u}_n + \Delta t\big(-A\boldsymbol{u}_{n+1} + \boldsymbol{f}(\boldsymbol{u}_n)\big) + G(\boldsymbol{u}_n)\,\Delta\boldsymbol{W}_n.$$

Chapter 9. **p372** In Example 9.1, "At each $x \in (0,1)$, $a(x)$ is a uniform random variable with mean $\mu$" should read "at each $x \in (0,1)$, $a(x)$ has mean $\mu$". In (9.4), $a(x)$ is a linear combination of $P$ independent uniform random variables, which for $P \geq 2$ is not itself uniformly distributed: already for $P = 2$ the density of a sum of two independent, non-degenerate uniform random variables is trapezoidal and not constant.

**p385** In (9.23), the bound should read

$$\mathbb{E}\big[E_{\rm MC}^2\big] \le K^2 Q^{-1},$$

with $K$ as defined in (9.24). The proof bounds $\mathbb{E}[E_{\rm MC}^2]$ by $Q^{-1}\|\tilde{u}_h\|^2_{L^2(\Omega,H_0^1(D))}$ and then applies Lemma 9.13 with $p = 2$, which gives $\|\tilde{u}_h\|_{L^2(\Omega,H_0^1(D))} \le K$, so the square is required. Corollary 9.23 is correct as printed: its proof concludes $L = K^2$, which follows from $\mathbb{E}[E_{\rm MC}^2] \le K^2Q^{-1}$ but not from the printed $KQ^{-1}$.

**p392** In the remark following Corollary 9.35, "If $d = 2$, we require $q > 4$" should read $q \ge 4$, consistent with the hypothesis $q \ge 2d$ of the corollary.

**p394** In the caption of Figure 9.9, the variance should read $\mathrm{Var}(a(\boldsymbol{x})) \approx 4.6708$ and not 8.0257. For the parameters stated in the caption ($\mu_k = 0$, $\sigma_k = 1$), the formula given in Example 9.39 yields $(\mathrm{e}^{\sigma_k^2} - 1)\,\mathrm{e}^{2\mu_k+\sigma_k^2} = \mathrm{e}^2 - \mathrm{e} \approx 4.6708$. The mean $\approx 1.7487 = 0.1 + \mathrm{e}^{1/2}$ in the same caption is correct.

**p398** In the proof of Theorem 9.48, the three occurrences of $u$ — twice in $|u(\cdot,\boldsymbol{y})|_{H^2(D)}$ and once in $|u|_{L^2_p(\Gamma,H^2(D))}$ — should read $\tilde{u}$. Assumption 9.47 bounds $|\tilde{u}|_{L^2_p(\Gamma,H^2(D))}$, so the final step of the proof requires the tilde.

**p407** In Example 9.57, the MATLAB command should call `fem_blocks` and not `fem_components`. Algorithm 9.3 defines `[K_mats,KB_mats,f_vecs,wB]=fem_blocks(...)`, and the fifteen arguments supplied in the example match that signature.

**p420** In (9.120), $\dfrac{dv^{k_i+1}}{dy^{k_i+1}}$ should read $\dfrac{d^{k_i+1}v}{dy_i^{k_i+1}}$.

**p421** The order of differentiation is likewise misplaced in (9.121), in the statement of Lemma 9.75 and in (9.122), which should read $\dfrac{\partial^{k_i+1}v}{\partial y_i^{k_i+1}}$, $\dfrac{\partial^{k_i+1}\tilde{u}}{\partial y_i^{k_i+1}}$ and $\dfrac{\partial^{k_i+1}v(\boldsymbol{x},\cdot)}{\partial y_i^{k_i+1}}$ respectively. In (9.121) the subscript on $y_i$ is also missing. The correct form appears in the final display of the proof of Lemma 9.75.

**p423** In §9.6, "the Galerkin matrix $A \in \mathbb{R}^{(J+J_b)\times(J+J_b)}$" should read $A \in \mathbb{R}^{J\times J}$, in agreement with the $Q$ linear systems "of dimension $J \times J$" in the same sentence. The matrices $K_0$ and $K_\ell$ occurring in $A = K_0 + \sum_{\ell=1}^P y_{r,\ell}K_\ell$ are indexed by $i, r = 1, \dots, J$ in (9.93)–(9.94); the interior–boundary coupling is carried by the separate matrices $K_{\ell,\mathrm{B}} \in \mathbb{R}^{J_b\times J}$, which do not appear in this expression for $A$.

**p425** In the Notes, in the display defining the kernel $K(x, y)$ for the one-dimensional homogenisation corrector, the two cases should be interchanged:

$$K(x,y) := \begin{cases} x(y-1/2), & 0 \le x \le y, \\ (x-1)(y-1/2), & y < x \le 1, \end{cases}$$

equivalently $K(x,y) = (\frac{1}{2} - y)(\mathbf{1}_{y<x} - x)$. As printed, $K(0,y) = -(y-\frac{1}{2})$ and so $\int_0^1 K(0,y)^2\,dy = \frac{1}{12} \ne 0$, contradicting both $d(0) = 0$ and the variance stated immediately below. With the kernel above, $\int_0^1 K(x,y)^2\,dy = \frac{1}{12}x(1-x)(8x^2 - 8x + 3)$ exactly, reproducing the stated $\mathrm{Var}(d(x))$.

**p426** In the Notes, "$[G_\ell]_{ij} = \langle \psi_\ell\psi_j, \psi_k\rangle_p$" should read $[G_\ell]_{jk} = \langle \psi_\ell\psi_j, \psi_k\rangle_p$.

**p427** In the Notes, in the display following (9.130), $u_{ik}$ should read $u_{kj}$. Setting $\boldsymbol{x}_* = \boldsymbol{x}_k$ and using $\phi_i(\boldsymbol{x}_k) = \delta_{ik}$ collapses the double sum in (9.130) to $\sum_{j=1}^Q u_{kj}\,\psi_j(\boldsymbol{y})$.

Chapter 10. **p439** In Example 10.9, the eigenfunctions should read

$$\chi_j(x) = \sqrt{2/a}\,\sin(j\pi x/a),$$

as in Example 10.10. As printed, $\sqrt{2/a}\sin(j\pi x)$ is not an eigenfunction of $Au = -u_{xx}$ on $(0,a)$ with $\mathcal{D}(A) = H^2(0,a) \cap H_0^1(0,a)$ unless $a = 1$.

**p440** In the caption of Algorithm 10.1, the value should read $\epsilon = 0.001$, to agree with `myeps=0.001` in the listing below it (and in `get_oned_bj.m`, Algorithm 10.3).

**p441** In the caption of Algorithm 10.2, the first sentence should read "If `iFspace=0`, the output `dW` is a matrix of $M$ columns with $k$th entry $W^{J-1}(t+\kappa\Delta t_{\rm ref}, x_k) - W^{J-1}(t, x_k)$ for $k = 1, \dots, J-1$." In the listing, `iFspace=1` returns the coefficients $b_j\xi_j$ and the `else` branch, `iFspace=0`, returns `dst1(X)`; Example 10.10 accordingly calls `get_onedD_dW(bj,kappa,0,1)` to generate the sample paths of Figure 10.5.

**p441** In (10.18), both exponents should read $e^{2\pi i j(k-1)/J}$. With $x_k = a(k-1)/J$, the preceding line carries $e^{2\pi i \ell x_k/a} = e^{2\pi i \ell(k-1)/J}$; the printed $e^{ijk/J}$ drops the factor $2\pi$ and replaces $k-1$ by $k$. Only the corrected form is the inverse discrete Fourier transform named immediately below, as computed by `ifft` in `get_oned_dW.m`.

**p442–469** `meshgrid` is misused in Examples 10.12 and 10.40 and in Algorithms 10.5 and 10.10. See `exa_10.40.m`. In consequence Figure 10.11 was computed not from the stated initial data $u_0 = \sin(x_1)\cos(\pi x_2/8)$ but from $\sin(8x_1/\pi)\cos(\pi^2 x_2/64)$, which is likewise not periodic on $(0, 2\pi) \times (0, 16)$. The online code now uses the stated data, and the stated $J_1 = J_2 = 128$, and therefore no longer reproduces the printed figure.

**p451** In the proof of Theorem 10.26, the contraction condition should read

$$T^2 + K^2 T^{\zeta}/\zeta < \frac{1}{2L^2},$$

and not $1/2L^2$. The preceding display gives $\|\mathcal{J}u_1 - \mathcal{J}u_2\|^2_{\mathcal{H}_{2,T}} \le 2\left(T^2 + K^2 T^{\zeta}/\zeta\right) L^2 \|u_1 - u_2\|^2_{\mathcal{H}_{2,T}}$, so a contraction requires $2\left(T^2 + K^2 T^{\zeta}/\zeta\right) L^2 < 1$.

**p453** In the proof of Lemma 10.27, the constant should read $C_{\mathrm{III}} = (C_1 + C_2)(1 + K_T)$, without the square. The bounds on $\mathrm{III}_1$ and $\mathrm{III}_2$ add to give $(C_1 + C_2)(t_2 - t_1)^{\theta_1}\left(1 + \sup_{0 \le s \le T} \|u(s)\|_{L^2(\Omega,H)}\right)$, and (10.36) gives $1 + \sup_{0 \le s \le T} \|u(s)\|_{L^2(\Omega,H)} \le (1 + K_T)\left(1 + \|u_0\|_{L^2(\Omega,H)}\right)$; the display that follows uses $C_{\mathrm{III}}$ linearly.

**p453** In the proof of Lemma 10.27, the first line of the estimate of $\mathrm{III}_2'$ should read $\int_{t_1}^{t_2}$ in place of $\int_0^t$, since $\mathrm{III}_2' = \int_{t_1}^{t_2}\left(e^{-A(t_2-s)} - I\right) G(u(s))\, dW(s)$ and the Itô isometry integrates over $[t_1, t_2]$, as on the line that follows.

**p457** The covariance identity should read

$$\mathrm{Cov}\Big(W^J(t, x_i), W^J(t, x_k)\Big) = (t/h)\,\delta_{ik}, \qquad i, k = 1, \dots, J-1,$$

which is the index set used for $\boldsymbol{W}^J(t)$ immediately below. At $i = k = J$ the stated identity fails: $x_J = a$, so $W^J(t, x_J) = 0$ and the covariance is 0, not $t/h$.

**p458** In Example 10.31, the three values quoted as the errors, $1.562 \times 10^{-3}$, $3.906 \times 10^{-4}$ and $9.766 \times 10^{-5}$ for $N = 64$, 256, 1024, are the time steps $\Delta t = T/N = 0.1/N$; the errors $|v(i) - 0.3489|$ from the computation displayed above should be reported in their place. As printed the values fall by exactly a factor 4 as $N$ quadruples, so they fit $\Delta t^1$ and are inconsistent with both the quoted $0.96\,\Delta t^{0.65}$ and the order-$\Delta t^{1/2}$ rate stated next.

**p465** In (10.54), the definition should read

$$\mathcal{G}(s; u) := G(u) + G'(u)\, G(u) \int_{t_n}^{s} dW(r), \qquad t_n \le s < t_{n+1}.$$

The Milstein method (10.53) contributes $G(\tilde{u}_n)\Delta W_n + G'(\tilde{u}_n) G(\tilde{u}_n) I_{t_n, t_{n+1}}$, which the preceding display writes as $\int_{t_n}^{t_{n+1}}\left[G(\tilde{u}_n) + G'(\tilde{u}_n) G(\tilde{u}_n) \int_{t_n}^{s} dW(r)\right] dW(s)$; matching $\int_{t_n}^{t_{n+1}} \mathcal{G}(s; \tilde{u}_n)\, dW(s)$ in (10.48) forces the factor $G(u)$, and the proof of Lemma 10.36 uses the corrected form.

**p466** In the proof of Lemma 10.36, the norm should read

$$\|(G(u(s)) - \mathcal{G}(s, u(t_k)))\,\chi\|_{L^2(\Omega,H)} \le \|R\|_{L^2(\Omega,H)} \|\chi\|_\infty,$$

as in the display that follows: for $\chi \in C(\overline{D})$ the quantity $(G(u(s)) - \mathcal{G}(s, u(t_k)))\,\chi$ takes values in $H$, not in $\mathcal{L}(H)$.

**p471** In (10.62), the first term on the right-hand side should read $e^{-\Delta t \lambda_j}\, \widehat{u}_{j,n}$, with the indices in the order $j, n$ as on the left-hand side.

**p473** In the paragraph preceding Algorithm 10.12, the sentence "As $J = J_w$, $G_h(\boldsymbol{u}_{h,n})$ is a diagonal matrix with entries $g(u^k_{h,n})$" should read: multiplication by $g$ is diagonal (pointwise), and $G_h(\boldsymbol{u}_{h,n})\Delta \boldsymbol{W}_n$ is the $L^2(0, a)$ projection onto $V^h$ of that product, with $j$th entry $\langle g(u_{h,n})\Delta W_n, \phi_j\rangle_{L^2(0,a)}$. By the definition given below (10.67), the $j, k$ entry of $G_h(\boldsymbol{u}_{h,n})$ is $\langle G(\boldsymbol{u}_{h,n})\chi_k, \phi_j\rangle_{L^2(0,a)}$, which is not diagonal; Algorithm 10.12 accordingly assembles this term with the load-vector routine `oned_linear_FEM_b.m`.

**p480** In Exercise 10.8, the hypothesis should read $2\beta \le 0$ (that is, $\beta \le 0$), which is the range in which the bound is used in the proof of Theorem 10.26. For $0 < \beta \le 1/2$ the stated bound fails: as $A^{\beta}\mathrm{e}^{-tA}$ is diagonal, $\|A^{\beta}\mathrm{e}^{-tA}\|^2_{\mathcal{L}(H)} = \sup_j \lambda_j^{2\beta}\mathrm{e}^{-2t\lambda_j}$, and for $\beta > 0$ the maximiser of $\lambda \mapsto \lambda^{2\beta}\mathrm{e}^{-2t\lambda}$ is $\lambda = \beta/t$, not $\lambda_1$. For $Au = -u_{xx}$ on $(0,\pi)$ with homogeneous Dirichlet boundary conditions ($\lambda_j = j^2$, $\lambda_1 = 1$) and $\beta = 1/2$, taking $j = \lceil (2t)^{-1/2} \rceil$ gives $\|A^{1/2}\mathrm{e}^{-tA}\|^2_{\mathcal{L}(H)} \ge \mathrm{e}^{-4}/(2t)$ for $0 < t \le 1/2$, so the integral diverges while the claimed bound is $\frac{1}{2}\lambda_1^0 = \frac{1}{2}$.

Appendix.

**p483** In §A.2, the trapezium rule on a single interval should read

$$\int_a^b u(x)\,dx \approx \frac{b-a}{2}\big(u(a) + u(b)\big),$$

and not $\frac{1}{2(b-a)}\big(u(a) + u(b)\big)$. This is the integral of the linear interpolant $p(x) = \big(u(a)(b-x) + u(b)(x-a)\big)/(b-a)$ given in the preceding line.

**p487–488** In §A.5, the value of the Euler constant should read "Its value is approximately 0.5772" and not 0.5572, as $\gamma = 0.57721566\ldots$

**p487** The discrete Gronwall inequality (Lemma A.14) is incorrect. It should either be

**Lemma 1.** *Consider $z_n \ge 0$ such that $z_n \le a + bz_{n-1}$ for $n = 1, 2, \ldots$ and $a, b \ge 0$. If $b = 1$, then $z_n \le z_0 + na$. If $b \ne 1$, then*

$$z_n \le b^n z_0 + \frac{a}{1-b}(1 - b^n).$$

or

**Lemma 2.** *Consider $z_n \ge 0$ such that*

$$z_n \le a + b\sum_{k=0}^{n-1} z_k, \quad \textit{for } n = 0, 1, 2\ldots \tag{*}$$

*and constants $a, b \ge 0$. Then, $z_n \le a(1+b)^n \le a\exp(bn)$.*

The first lemma is (Stuart and Humphries, 1997, Theorem 1.1.12).

To prove the second lemma, notice it is true for $n = 0$. Assume it is true for $z_0, \ldots, z_{n-1}$. Then,

$$\begin{aligned}
z_n &\le a + b\sum_{k=0}^{n-1} z_k \quad \text{by (*)}\\
&\le a + b\sum_{k=0}^{n-1} a(1+b)^k \quad \text{by induction assumption}\\
&= a + ab\frac{1-(1+b)^n}{1-(1+b)} \quad \text{by geometric summation formula}\\
&\le a + ab\frac{1-(1+b)^n}{-b} = a + ab((1+b)^n - 1) = a(1+b)^n.
\end{aligned}$$

Therefore, $z_n \le a(1+b)^n$ for all $n = 0, 1, 2, \ldots$ by induction. Finally, $z_n \le a\exp(bn)$ as $1 + x \le \exp(x)$ for $x \ge 0$. The second lemma can be used in the proof of Theorems 3.55 and 10.34.